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"ON THE ORIGIN OF SOLAR SYSTEM"*

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ABSTRACT. The paper deals with the tidal theories of the origin of the Solar System. The author has shown in §2 to §5 that in *two-body problem* there is no possibility for the formation of the planetary ribbon, from which the present planets are supposed to have been developed, by a close encounter or a grazing collision between two stars of usual masses.

In §6 and the following sections, the author has examined mathematically the theory extended by Lyttleton to explain the origin of planets and has proved that in the most favourable situation for the formation of the planetary ribbon in the *three-body problem*, at the middle of the collision between the sun's companion and a visiting star, the sun would come so near to its companion and hence to the visiting star that a very close encounter or collision between them can hardly be avoided.

§1. From time to time, several theories have been suggested to account for the peculiar dynamical arrangement of bodies which constitute the solar system. But when these theories are tested in the light of the distribution of angular momentum in the system, none is found to bear the test and a new theory was expected to come forth which may explain this aspect of the solar system also. H. N. Russell, along with several other theories, suggested [1] "that our sun might have been a binary star having a companion much smaller than itself which had been revolving about the sun at a distance comparable with major planets, and that the collision between this body and a passing star broke this companion into fragments from which the present planets were developed." He gave up the hypothesis thinking that it was not possible to account satisfactorily for the ionisation of the companion from the sun and the ultimate formation of the terrestrial planets.

Lyttleton [2] tried to give a mathematical treatment to Russell's suggestion that the sun had once been a component of a binary star whose companion had been removed by a close encounter with a passing star. He also studied the particular case where the masses of these three stars were taken to be equal. He studied mainly the ionisation of the companion from the sun and assumed that "the mechanism which effects this disruption also produces the planets giving them widely varying angular momentum per unit mass and possibly causing them to proceed round the sun in the same general direction more or less in a plane."

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Luyten and Hill studied[3] in detail the two cases, *viz.*, where the masses of all the three stars are equal and where the intruding star was less massive than the sun. They pointed out that the two important factors were over-looked by Lyttleton, *viz.*, (1) kinetic energy required to form the planets is very large and the intruding star must have had an initial velocity of about 100 km./sec. at least relative to the sun at a great distance from the binary system which is of rare occurrence in nature and (2) that Lyttleton assumed that the whole of the planetary ribbon was available for the capture by the sun, whereas the calculations show that only 6% of the whole length of the ribbon could be captured if at all possible and, in view of these circumstances, the theory is untenable.

To avoid the requirements of very large velocities of the intruding star, Lyttleton[4] put forward a new hypothesis that the intruding star was much more massive than the sun. In a recent paper[5], Luyten has criticised this hypothesis of more massive intruder and suggested that it is exceedingly improbable that the sun could capture any part of the ribbon without itself being captured by the intruder, and that "in the situation most favourable to capture, the sun must have been running roughly parallel to the filament for some time and must have suffered a close approach or a collision with the intruder.

The whole problem is very complicated on account of the fact that it involves the consideration of the problem of three bodies in the beginning and after collision, a consideration of multiple bodies. The above-mentioned treatment by Lyttleton, Luyten and Hill, as it stands, lacks in Mathematical analysis. Lyttleton assumes that the sun remains from the beginning to the end of the encounter at the same mutual distance from the companion. In a recent paper, Luyten has criticised Lyttleton's vector diagram and pointed out the errors, but his criticism also is more or less speculative in character. In the present paper the author has tried to study the change in the velocity and position of the sun during this catastrophe from the mathematical point of view.

Since the time the tidal theory has been suggested, the possibility of a close encounter or a grazing collision resulting in the production of a material ribbon, from which the present planets were formed, was assumed. Before discussing the main problem, the validity of this assumption is tested.

§2. *Tidal Force* :—Let S_1 and S_2 be two stars whose elements are given below :—

	S_1	S_2
Mass	M_1	M_2
Undisturbed mean radius	$R_1 = \sqrt[3]{\dfrac{3M_1}{4\pi\rho_1}}$	$R_2 = \sqrt[3]{\dfrac{3M_2}{4\pi\rho_2}}$
Mean density	ρ_1	ρ_2.

Distance between S_1 and $S_2 = d$.

Let $M_2 = nM_1$, $d = qR_1$ and $r = pR_1$.

Also let ABCD (diagram I) be the section of S_1 by the relative orbital plane of S_1 and S_2, which we take to be the plane $\phi = 0$.

$$F_1 = \text{attraction at } P' \ (r, \theta, 0) \text{ due to } S_2$$

$$= \frac{M_2 G}{d^2 + r^2 - 2rd \cos\theta},$$

regarding S_2 to be spherical.

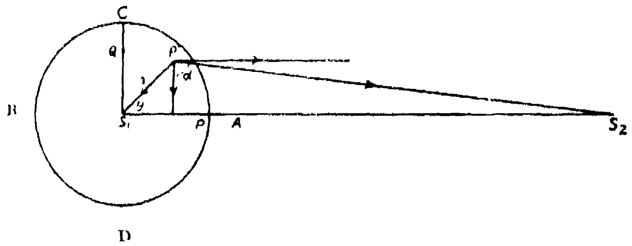


DIAGRAM I

$$F = \text{attraction at } S_1 \text{ due to } S_2 = \frac{M_2 G}{d^2},$$

$$\text{component of } F_1 \text{ parallel to } S_1 S_2 = \frac{M_2 G \ (d - r\cos\theta)}{(d^2 + r^2 - 2rd\cos\theta)^{\frac{3}{2}}} \qquad \ldots \quad (1)$$

$$\text{and component of } F_1 \text{ perpendicular to } S_1 S_2 = \frac{M_2 G \ (r \sin\theta)}{(d^2 + r^2 - 2rd\cos\theta)^{\frac{3}{2}}}. \qquad \ldots \quad (2)$$

Therefore the tide-generating force at P′ consists of two components: one parallel to S_1S_2 and equal to

$$\frac{M_2 G \ (d - r\cos\theta)}{(d^2 + r^2 - 2rd\cos\theta)^{\frac{3}{2}}} - \frac{M_2 G}{d^2}$$

and other perpendicular to $S_1 S_2$ and equal to

$$\frac{M_2 G \ r \sin\theta}{(d^2 + r^2 - 2rd\cos\theta)^{\frac{3}{2}}}$$

$$\text{Therefore } T = M_2 G \sqrt{\frac{1}{d^4} + \frac{1}{(d^2 + r^2 - 2rd\cos\theta)^2} - \frac{2(d - r\cos\theta)}{(d^2 + r^2 - 2rd\cos\theta)^{\frac{3}{2}}}} \quad (3)$$

which acts in a direction making an angle α with the perpendicular direction to S_1S_2, such that

$$\tan\alpha = \frac{d^2(d - r\cos\theta) - (d^2 + r^2 - 2rd\cos\theta)^{\frac{3}{2}}}{d^2 r \sin\theta} \qquad \ldots \quad (4)$$

When r/d is small we may put

$$T = \frac{M_2 G r}{d^3} \sqrt{1 + 3\cos^2\theta}$$

and
$$\tan\alpha = 2\cot\theta - \frac{3r}{2d} \sin\theta.$$

The component of T along $S_1 P' = T \sin(\alpha - \theta)$.

The force of gravity at P' due to $S_1 = \frac{M_1 G r \kappa}{R_1^3}$,

where κ is a function of θ (independent of r) depending on the degree of distortion of S_1.

The component of the tidal force balances the gravity at P'

if
$$T \sin(\alpha - \theta) = \frac{M_1 G r \kappa}{R_1^3},$$

i.e., if

$$M_2 G \sin(\alpha - \theta) \sqrt{\frac{1}{d^4} + \frac{1}{(d^2 + r^2 - 2rd\cos\theta)^2} - \frac{2(d - r\cos\theta)}{(d^2 + r^2 - 2rd\cos\theta)^{\frac{3}{2}}}} = \frac{M_1 G r \kappa}{R_1^3}$$

which may be put in the form

$$p = n\sin(\alpha - \theta) \sqrt{\frac{1}{q^4} + \frac{1}{(q^2 + p^2 - 2pq\cos\theta)^2} - \frac{2(q - p\cos\theta)}{(q^2 + p^2 - 2pq\cos\theta)^{\frac{3}{2}}}} \quad \ldots (7)$$

where
$$n = \frac{n}{\kappa} \text{ (function of } \theta) \quad \ldots (8)$$

and
$$\tan\alpha = \frac{q^2(q - p\cos\theta) - (q^2 + p^2 - 2pq\cos\theta)^{\frac{3}{2}}}{q^2 p \sin\theta}. \quad \ldots (9)$$

Let us now put $\theta = 0$. Then the points of equality of the tidal force and the gravitational force on $S_1 S_2$ are given by

$$\frac{1}{(q-p)^2} - \frac{1}{q^2} = \frac{p}{n}. \quad \ldots (10)$$

The three roots of (10) are

$$p_0 = 0 \quad \text{(centre)},$$

$$p_1 = q - \frac{n}{2q^2}\left\{1 - \sqrt{1 + \frac{4q^3}{n}}\right\},$$

$$p_2 = q - \frac{n}{2q^2}\left\{1 + \sqrt{1 + \frac{4q^3}{n}}\right\}.$$

Let $2n > q^3$.

Then, if $2n > 4q^3$, one root is positive and the other negative ; the negative root is less than the positive one in magnitude ; if $2n < 4q^3$, the negative root is greater than the positive root in magnitude.

Let $2n < q^3$. Then both the roots are positive.

We shall now consider the variation in the difference of the tidal force and gravitational force along the radious of S_1 directed towards $S_2°$. Let

$$y = M_2G\left[\frac{1}{(d-r)^2} - \frac{1}{d^2}\right] - \frac{M_1G\gamma\kappa}{R_1^3}.$$

Putting $\frac{M_1Gk}{R_1^2} = g$ and then $y = g\,z$, we have

$$Z = n\left[\frac{1}{(q-p)^2} - \frac{1}{q^2}\right] - p.$$

Differentiating with respect to p we have

$$\frac{dz}{dp} = \frac{2n}{(q-p)^3} - 1.$$

Hence z increases with p if $2n > (q-p)^3$ and decreases with the increasing p if $2n < (q-p)^3$. These inequalities are all the more satisfied if $2n > q^3$ and $2n < (q-p_1)^3$ respectively, where p_1 is the greatest value of p.

The complete tidal disruption of S_1 would be possible only in the first case, *i.e.*, when $2n > q^3$ because in that case the tidal force on the material everywhere on the axis would be directed outwards ; whereas in the case $2n < (q-p_1)^3$, there cannot be even the slightest shedding of material from S_1. For the partial shedding-off of the material from S_1 the value of n should be between $\frac{1}{2}q^3$ and $\frac{1}{2}(q-1.65396)^3$, where p_1 has been taken equal to 1.65396 corresponding to the critical [6] eccentricity .882579 required for the disruption of S_1. In the following tables the limiting values for n have been collected for different values of q.

TABLE I

If S_1 is in form of a sphere			If S_1 is a critical spheroid	
$n <$	$n >$	q	$n <$	$n >$
4	.5	2	1.947	.0101
32	13.5	4	15.578	3.143
108	62.5	6	52.574	19.980
256	171.5	8	124.621	62.206
500	364.5	10	243.400	141.501
864	665.5	12	420.595	269.552
1462	1098.5	14	711.702	458.040

In these calculations we have put $p_1 = 1$ in case of the spherical star

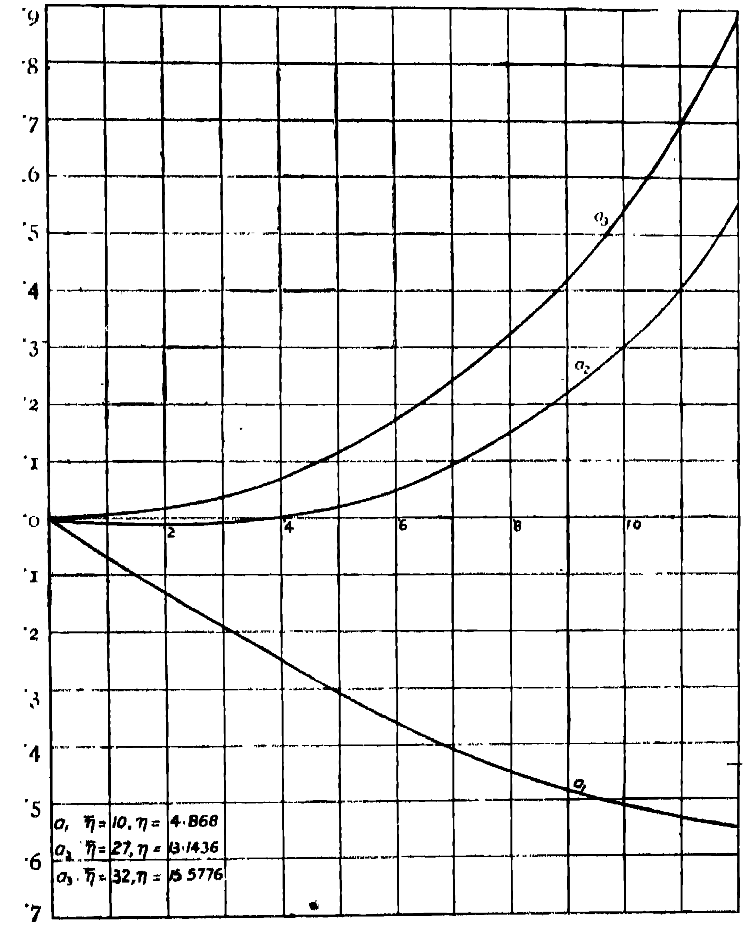
and $\quad p_1 = 1.65396$ and $k = \frac{3}{2} \cdot \frac{1 - e^2}{e^3} \cdot \left[\log \frac{1+e}{1-e} - 2e \right]^{-1}$

$= .4868$, taking $e = .8826$.

TABLE II

n	n	$p=0$	.2	.4	.6	.8	1.0	1.2
10	4.868	$z=0$	.1325	−.2534	−.3590	−.4485	−.5139	−.5490
27	13.1436	0	−.0178	−.0042	.0483	.1491	.3125	.5577
32	15.5776	0	.0160	.0691	.1683	.3248	.5555	.8832

In the graph I curves have been drawn for three values of n showing the behaviour of z as p is increased, taking $q = 4$.



GRAPH I

When the star is the spheroid $\frac{x^2}{a^2} + \frac{y^2+z^2}{b^2} = 1$ $(a > b)$, then $k = +\frac{3}{2} R_1{}^3 \int_0^\infty \frac{du}{(a^2+u)^{\frac{3}{2}}(b^2+u)}$

$$= \frac{3}{2} R_1{}^3 \cdot \frac{1}{a(a^2-b^2)^{\frac{3}{2}}} \cdot \left[a \log \frac{a + \sqrt{a^2-b^2}}{a - \sqrt{a^2-b^2}} - 2\sqrt{a^2-b^2} \right] = \frac{3}{2} \cdot \frac{1-e^2}{e^3} \cdot \left[\log \frac{1+e}{1-e} - 2e \right].$$

In a_3, since at every point along S_1A the tidal force is greater than the gravity, the acceleration at every point would be directed away from the centre S_1 and the stellar material would move outward resulting in ejection, because the action of the fluid pressure is always to encourage the ejection. In a_2, up to $p=.418$ the gravity is greater than the tidal force and after that the latter is greater than the former and thus the acceleration at every point corresponding to greater values of p than .418 would be directed away from S_1 and hence a partial ejection may take place. In a_1 ejection does not seem possible.

It should be noted that if S_1 is in the form of a sphere, the same curves correspond to the masses $n=10$, 27 and 32. Thus, if initially S_1 is undistorted, for ejection of the material, S_2, which is supposed to be situated at the same distance $q=4$, should possess comparatively larger masses.

We shall now consider the variation in magnitude and direction of the tidal force acting along an imaginary circular section of S_1 in the relative orbital plane of S_1 and S_2. We shall make use of the approximate formula, since in that case, too, the fundamental characteristics remain unaltered. Formulae (5) and (6) are :

$$T=\frac{M_2Gr}{d^3}\sqrt{1+3\cos^2\theta} \qquad \dots \quad (5)$$

and

$$\tan a=2\cot\theta-\frac{3}{2}\cdot\frac{r}{d}\cdot\sin\theta. \qquad \dots \quad (6)$$

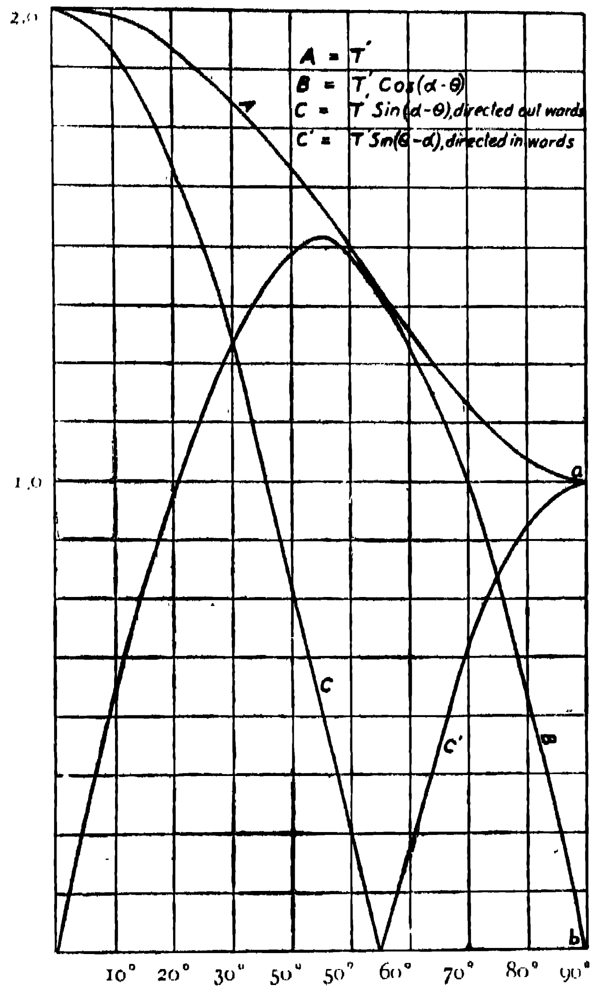
Instead of plotting T against θ, we shall plot T′ against θ, where

$$T'=\frac{Td^3}{M_2Gr}.$$

The components of T′ tangential to the circle and along the radius vector are equal to T′ $\cos(\theta-a)$ and T′ $\sin(\theta-a)$. We shall plot these two components also in the same graph (see graph II).

TABLE III

θ	T′	T′ cos $(a-\theta)$	T′ sin $(\theta-a)$	
0°	2	0	2	Away from the centre.
10°	1.98	.513	1.91	
20°	1.91	.963	1.65	
30°	1.80	1.30	1.25	
40°	1.66	1.48	750	
50°	1.50	1.48	.240	
60°	1.32	1.30	.230	Towards the centre.
70°	1.16	.963	.647	
80°	1.04	.513	.905	
90°	1.00	.000	1.000	



GRAPH II

In preparing the above table we have made use of the formula $\tan \alpha = 2 \cot \theta$ instead of the formula (6). If the formula (6) would have been employed then the curve B would not reach the θ-axis, it would have been much above it for $\theta = 90°$ and similarly c' would not join A at $\theta = 90°$. The end points a and b would move towards one another along ab as we consider more and more correct approximate form of the formula (5) and (6).

§3. *Material required to form the ribbon* :—Considering the stars defined in §2, we have

$$M_1 = \text{the mass of } S_1 = \tfrac{4}{3}\pi R_1^3 \rho_1 .$$

As mentioned in §1, the total mass† of the planets $= .0015\ M_{\odot}$ and that only 6% of the filament by length may be captured in form of the primitive planets if at all. Therefore the total mass that should be ejected from S_1 to form the

† Russell, Dugan and Stewart's "Astronomy," part (i), appendix p. (H).

ary ribbon $=.025\ M_{\odot}$, if the mean density of the filament is also equal to the mean density of the star S_1. In general the density of the filament would be much less than that of the parent star, firstly because the material forming the filament would be from the boundary of the star, secondly under the opposing gravitational forces of the two encountering stars the filament would diffuse in the space rapidly and thirdly because the temperature of the filament would be at least few thousand degrees centigrade which would help immensely the diffusion pointed just above. If S_1 possesses mass equal to that of the sun, then the mass of the material that should be ejected out should be equal to $.025\ M_1$

$$= .025 \times \tfrac{4}{3}\pi R_1^3 \rho_1.$$

This material would form a sphere of radius R_0 and mean fluid density ρ_1,

where $$R_0 = .2924\ R_1$$

and hence p_0 must be nearly 1.0692 if S_1 is a critical spheroid or .4152 if the star is spherical, where p_0 is the value of p where $z=0$—of course positive and other than zero. For this to be realised, the masses of S_2 for values of q are obtained in the following table :—

TABLE IV

When S_1 is a sphere		When S_1 is a critical spheroid
n	q	n
2.7843	2	.5757
27 109	4	9 6444
96.923	6	38.979
236 24	8	100.23
469.10	10	205.15
819.42	12	365.26
1311 3	14	592.39

Taking S_1 equal to the sun we find that in order that there may be a possibility for the material, necessary to form the planetary ribbon, to be ejected out, the mass of S_2 for a particular value of q lies between the two limits given in the above table for S_1 would neither be a sphere nor a critical spheroid in general,

§4. *Tidal distortion of a spherical star into a critical spheroid* :—Putting $\theta=\pi/2$ in (3), we get the tidal force at a point like Q. We have at Q

$$T=M_2G\sqrt{\frac{1}{d^4}+\frac{1}{(d^2+r^2)^2}-\frac{2}{d(d^2+r^2)^{\frac{3}{2}}}}$$

and
$$\tan\alpha=\frac{d^3-(d^2+r^2)^{\frac{3}{2}}}{d^2r}\qquad\text{(from 4).}$$

Hence
$$T\cos\alpha=M_2G/(d^2+r^2)^{\frac{3}{2}}$$
$$T\sin\alpha=\frac{M_2G\{d^3-(d^2+r^2)^{\frac{3}{2}}\}}{d^2(d^2+r^2)^{\frac{3}{2}}}.$$

While considering the tidal deformation of S_1 we shall regard S_2 a point mass. Let at any stage of deformation a, b and b be the lengths of the axes of the spheroid to which the star has been deformed under the influence of S_2.

Pressure at S_1 as obtained from the consideration of the equilibrium of the fluid column along S_1A

$$=\int_0^a\left[\left\{\frac{-2\pi G\rho_1ab^2}{a(a^2-b^2)^{\frac{3}{2}}}\left(2\sqrt{a^2-b^2}+a\log\frac{a-\sqrt{a^2-b^2}}{a+\sqrt{a^2-b^2}}\right)\right\}-M_2G\left\{\frac{1}{(d-r)^2}-\frac{1}{d^2}\right\}\right]dr$$

$$=-\frac{\pi G\rho_1a^2b^2}{(a^2-b^2)^{\frac{3}{2}}}\left\{2\sqrt{a^2-b^2}+a\log\frac{a-\sqrt{a^2-b^2}}{a+\sqrt{a^2-b^2}}\right\}-M_2G\left[\frac{1}{(d-a)}-\frac{1}{d}-\frac{a}{d^2}\right],$$

where the expression within the serpentine brackets arises due to the attraction of the star S_1 and the remaining expression due to the tidal force of S_2.

Similarly the pressure at S_1 as obtained from the consideration of the equilibrium of the fluid column S_1B

$$=\pi G\rho_1ab^4\left[\frac{1}{ab^2}+\frac{1}{2a(a^2-b^2)^{\frac{3}{2}}}\left\{2\sqrt{a^2-b^2}+a\log\frac{a-\sqrt{a^2-b^2}}{a+\sqrt{a^2-b^2}}\right\}\right]$$
$$+M_2G\left[\left\{\frac{1}{d}-\frac{1}{\sqrt{d^2+b^2}}\right\}\right].$$

These two pressures must be equal. Hence

$$M_2\left[\frac{a}{d^2}-\frac{1}{d-a}+\frac{1}{\sqrt{d^2+b^2}}\right]$$
$$=\frac{3}{4}\frac{M}{a}\left[1+\frac{2a^2+b^2}{2(a^2-b^2)^{\frac{3}{2}}}\left\{2\sqrt{a^2-b^2}+a\log\frac{a-\sqrt{a^2-b^2}}{a+\sqrt{a^2-b^2}}\right\}\right].$$

Now $\qquad a=R_1(1-e^2)^{-\frac{1}{3}}$ and $b=R_1(1-e^2)^{\frac{1}{6}}$.

According to the notations given in §2, we can put this equation in the form

$$n\left[\frac{1}{q^2(1-e^2)^{\frac{1}{3}}}-\frac{1}{q-(1-e^2)^{-\frac{1}{3}}}+\frac{1}{\sqrt{q^2+(1-e^2)^{\frac{1}{3}}}}\right]$$

$$=\frac{3}{4}(1-e^2)^{\frac{1}{3}}\left[1-\frac{3-e^2}{2e^3}\left\{2e+\log\frac{1-e}{1+e}\right\}\right]. \quad \ldots \quad (11)$$

When S_2 is at S''_2, the position of S_2 of its nearest approach to S_1 or much before this (such that S_1S_2 is of the order of 20 to 30 solar radii) in the diagram (ii), S_1 must be so much deformed that its eccentricity is greater than the critical value .882579, otherwise there is no chance for ejection of the material (see in §5). In the case when S_1 is deformed to the critical shape the material would be shed away from the star S_1 irrespective of the motion of S_2 on account of the momentum imparted to it by the encounter of these stars We shall now treat the equation (11) numerically. Taking $e=.8826$, we have

TABLE V

$q =$	2	4	6	8	10	12	14	20	30
$n =$	.0,8	2.578	10 557	27.205	55.66	99.1	160 7	484.7	1682

The above table gives the relation between q and n. In order that the spherical star S_1 may be deformed in the form of a spheroid of eccentricity $e=.8826$ due to the tidal action of another star S_2 situated at the distance determined by q, the mass of the latter must be n. It is quite evident from the table (V) that in order that the requisite deformation ($e=.8826$) may be realized from a distance of about 20 to 30 solar radii the mass of S_2 must be unusually enormous.

In §2, §3 and §4 we have considered the tidal distortion of a spherical star and the conditions for its ultimate disruption when the visiting star is stationary so that the tidal forces have got enough time to redistribute the material of S_1 and produce the calculated distortion. It has been shown above that in order to deform S_1 (assumed to be sperical initially) so as to reduce it to the critical spheroid ($e=.8826$) the stationary star situated at the distance of 20 solar radii must possess 485 times the mass of the sun. Similarly, if we take the distance between S_1 and S_2 to be 30 solar radii, the requisite mass of S_2 must be 1682 times the solar mass. By taking M_1 greater than M_0 we shall be able to increase the value of p_0. ; but on account of the increased mass of S_1 we shall require more massive intruder in order to balance the gravitational force of S_1 at that distance even. In order that the tidal force may be greater than the gravitational force on the material of S_1 necessary to form the planetary ribbon when S_2 is at a distance of 8 solar radii, the latter must be about 100 times more massive than the sun if S_1 is already deformed to the critical eccentricity and about 236 times more massive than the sun if S_1 is spherical. The reduction of the distance between S_1 and S_2 and consequently reducing q, no doubt, reduces the value for the mass of S_2, but on account of the extensions of the stars there is bound to be a very tremendous collision between them in the first place and in the second it will be seen in the discussion of the dynamical problem below that the duration of the effective encounter and the collision is so much reduced by reducing q that the disruption becomes an impossibility unless there is very tremendous collision between the stars and in that case what actually happens cannot be predicted. Thus it can be inferred that in statical problem in order to ensure disruption of S_1 by the tidal action of S_2, situated at suitable distance, the latter must possess unusually great mass.

§5. *Stellar encounter*:—Nearly everybody has assumed the possibility of encounter between two stars resulting in the formation of the planetary ribbon. We shall in the very beginning make an attempt to study if it is at all possible to produce the desired ribbon by a close encounter or a grazing collision between two stars of usual masses and sizes if we take the relative motion of the stars into account.

As we can conclude from the discussions in §2, the greatest tidal forces are excited on S_1 when the passing star is nearest to S_1 and that seems to be the most favourable situation for the removal of the material from the former, in the form of planetary ribbon. It will be seen in the following treatment that the time during which the grazing collision lasts is extremely small. Is it then possible even for those maximum tidal forces which act for a very short time at a particular point of the material of S_1 to produce the desired ribbon ? The following treatment shows that the answer is in the negative.

Before proceeding further, it is necessary to define the encounter and its duration. By encounter we mean the close approach of a star within a certain specified distance of another star and the duration of the encounter may then be conveniently defined as the time during which the intruding star is within that specified distance.

In §9, we have calculated the eccentricity of the hyperbolic orbit of the intruding star relative to the other star at least in two cases. In both these cases we find that e is very nearly equal to unity (*viz.*, 1.068 and 1.014). This is quite in keeping with the general consideration of the stellar motions. The stars possess very small *proper motions* but if then any sort of collision is to take place the intruder moves towards the star under its gravitational force and the orbit described under these circumstances must be more or less a parabola. Hence in general we shall assume that during the encounter between the stars, S_2 is describing a parabola under the action of the gravitational force of S_1.

Let us assume that the eccentricity of the hyperbolic orbit described by S_2 relative to S_1 is e. Then the equation to the orbit will be

$$r = \frac{\dfrac{h^2}{M_1 + M_2}}{1 + e\cos\theta}, \qquad \text{.. (12)}$$

where h is the areal constant determined later on. At any instant

$$\text{velocity} = \sqrt{\dot{r}^2 + r^2\dot{\theta}^2}$$

$$= \sqrt{\frac{2(M_1+M_2)}{r} + \frac{(M_1+M_2)^2(e^2-1)}{h^2}}. \qquad \text{... (13)}$$

If the hyperbolic motion of S_2 relative to S_1 begins when the former is at a distance l_1 from the latter and the velocity of S_2 relative to S_1 at that instant be $\dot{R}_b$, then we have

$$\dot{R}_b^2 = \frac{2(M_1+M_2)}{l_1} + \frac{(M_1+M_2)^2(e^2-1)}{h^2} \qquad \text{... (14)}$$

and

$$\frac{h^2}{M_1+M_2} = \frac{(M_1+M_2)(e^2-1)}{\dot{R}_b^2 - \dfrac{2(M_1+M_2)}{l_1}} \qquad \text{... (15)}$$

hence (12) can be written as

$$r = \frac{\dfrac{(e^2-1)(M_1+M_2)}{\dot{R}_b^2 - \dfrac{2(M_3+M_2)}{l_1}}}{1 + e\cos\theta}. \qquad \text{... (16)}$$

Let the shortest distance between S_1 and S_2 be d_1. Then

$$d_1 = \frac{(e-1)(M_1+M_2)}{\dot{R}_b - \dfrac{2(M_1+M_2)}{l_1}} \quad (\theta = 0). \qquad \text{.... (17)}$$

Therefore

$$e = 1 + \frac{d_1}{M_1+M_2}\left\{\dot{R}_b^2 - \frac{2(M_1+M_2)}{l_1}\right\} \qquad \text{... (18)}$$

and

$$r = \frac{(e+1)d_1}{1 + e\cos\theta}; \qquad \text{... (19)}$$

also

$$\dot{R}_b^2 = \frac{2(M_1+M_2)}{r} + \left\{\dot{R}_b^2 - \frac{2(M_1+M_2)}{l_1}\right\}$$

$$= \frac{2(M_1+M_2)}{r} + \frac{(M_1+M_2)(e-1)}{d_1}, \qquad \text{... (20)}$$

where R_b is the velocity of S_2 relative to S_1 at any instant when the former is moving on the hyperbola (19).

As long as the star S_2 is moving towards S_1, it will move on the hyperbola specified by (19) but when S_2 is at the shortest distance from S_1 there may be exchange of material in the case of the grazing collision and in case of an encounter the tidal forces may alter the velocity and the motion may be modified. Hence we assume that the hyperbolic orbit (19) ceases at the perihelion of (19) and S_2 proceeds along a different hyperbola. If we futher assume that this modified hyperbolic motion ends at a distance l_2 from S_1 when S_2 is moving with a velocity $\bar{R}$ relative to the former, the equation to the orbit will be

$$r=\frac{(e_1+1)d_1}{1+e_1\cos\theta} \qquad \ldots \quad (21)$$

and

$$R_a^2=\frac{2(M_1+M_2)}{r}+\left\{R_a^2-\frac{2(M_1+M_2)}{l_2}\right\}$$

$$=\frac{2(M_1+M_2)}{r}+\frac{(M_1+M_2)(e_1-1)}{d_1}, \qquad \ldots \quad (22)$$

where e_1 is the eccentricity of the new orbital hyperbola and R_a the velocity of S_2 relative to S_1 at any instant on it.

Hence the loss of the kinetic energy *

$$=\frac{1}{2}\frac{M_1M_2}{M_1+M_2}\left(\overline{R}_b^2-\overline{R}_a^2\right)+M_1M_2\left(\frac{1}{l_2}-\frac{1}{l_1}\right). \qquad \ldots \quad (23)$$

This kinetic energy is supposed to be utilised in the formation of the planetary ribbon. But later on it will be seen that a major part of this energy is utilised in accelerating the motion of M_0 in the three-body problem dicussed in §6 and the following sections.

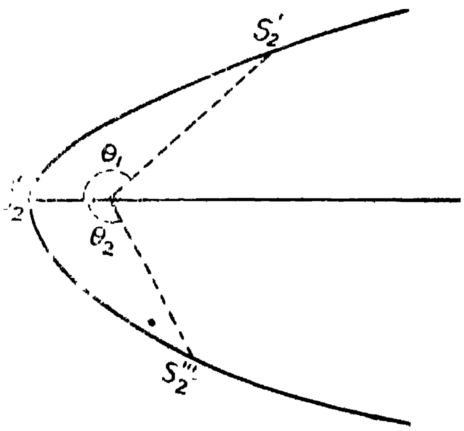


DIAGRAM II

* It may be noted here that we have an additional term in this expression which vanishes when $l_1=l_2$.

In the diagram II when S_2 is at S_2'' it is nearest to S_1 so that $S_1 S_2'' = d_1$. $S_2' S_2''$ and $S_2'' S_2'''$ are the branches of the two hyperbolas assumed in (19) and (21) where S_2' and S_2''' are the positions of S_2 where its hyperbolic motion relative to S_1 begins and ends respectively. Let $\overline{T}$ be the total time during which the hyperbolic motion lasts. Then

$$\overline{T} = \frac{d_1^{\frac{3}{2}}}{(M_1+M_2)^{\frac{1}{2}}}\left[(e+1)^{\frac{3}{2}}\int_0^{\theta_1}\frac{d\theta}{(1+e\cos\theta)^2}+(e_1+1)^{\frac{3}{2}}\int_0^{\theta_2}\frac{d\theta}{(1+e_1\cos\theta)^2}\right],$$

where
$$\theta_1 = \cos^{-1}\left[\frac{1}{e}\left\{\frac{(e+1)d_1}{l_1}-1\right\}\right]$$

and
$$\theta_2 = \cos^{-1}\left[\frac{1}{e_1}\left\{\frac{(e_1+1)d_1}{l_2}-1\right\}\right].$$

Now to simplify the calculations if we assume that $l_1 = l_2 = l$ (say), $e = e_1 = 1$ (the justification of this assumption is given above), we have

$$T' = \frac{2\sqrt{2}}{3}\sqrt{\frac{l-d_1}{M_1+M_2}}\,(l+2d_1), \qquad \ldots \quad (24)$$

where T' is the total time during which S_2 is within the distance l of S_1. Similarly the time T'' taken by S_2 to move from one end of the latus-rectum to the other of the parabolic orbit will be given by

$$T'' = \frac{8\sqrt{2}}{3}\cdot\frac{d_1^{\frac{3}{2}}}{\sqrt{M_1+M_2}}; \qquad \ldots \quad (25)$$

also the time taken by S_2 to move from one end of the latus rectum to any position (r, θ) will be given by

$$T = \frac{\sqrt{2}}{3}\cdot\frac{1}{\sqrt{M_1+M_2}}\cdot\left[(2d_1+r)\sqrt{r-d_1}\;-4d_1^{\frac{3}{2}}\right]; \qquad \ldots \quad (26)$$

Case I : Let $M_1 = M_2 = M_\odot$,

$l_1 = 10$ astronomical units,

$d_1 = 1/40$,, ,, ,

and $r = 1/2$,, ,, .

Then $T' = 3{\cdot}5$ years (approx.),

$T'' = 15$ hours (,,),

and $T = 169$ hours (,,).

Case II : Let $M_1 = 2M_\odot$, $M_2 = 8\,M_\odot$,

$l = 10$ astronomical units,

$d_1 = 1/40$,, ,, ,

and $r = 1/2$,, ,, .

Then $T' = 1\cdot5$ years (approx.)

$T'' = 6\cdot5$ hours (,,),

and $T = 76$ hours (,,).

We shall refer these cases as case I and case II everywhere in the following treatment.

In the case of the hyperbolic motions, the time would be comparatively less. Also as we increase the masses of the stars the time decreases rapidly. As a matter of fact, T varies inversely as the square root of the sum of the masses of the two stars.

We shall now study the phenomenon of the production of the tides in S_1 under the action of S_2. Let us now imagine that the latter is at a distance d from the former. In such a position the heights h_1 and h_2 of the tides in S_1 at the points nearest and farthest from S_2 would be given by [7]

$$h_1 = \frac{M_2}{M_1} \cdot \frac{R_1^4}{d^3} \cdot \left(\Delta_1 + \frac{R_1}{d} \Delta_3 + \frac{R_1^2}{d^2} \Delta_4 \right) \quad \ldots \quad \ldots \text{ at the nearest pole,}$$

and

$$h_2 = \frac{M_2}{M_1} \cdot \frac{R_1^4}{d^3} \cdot \left(\Delta_2 - \frac{R_1}{d} \Delta_3 + \frac{R_1^2}{d^2} \Delta_4 \right) \ldots \quad \ldots \text{ at the farthest pole,}$$

where Δ's are certain numerical constants.

Similarly the maximum contraction in S_1 will be given by

$$h_3 = \tfrac{1}{2} \cdot \frac{M_2}{M_1} \cdot \frac{R_1^4}{d^3} \cdot \left(\Delta_2 - \tfrac{3}{4} \Delta_4 \frac{R_1}{d} \right) \quad \ldots \quad \text{at the equator.}$$

To the first approximation we may put

$$h_1 = h_2 = \frac{M_2}{M_1} \cdot \frac{R_1^4}{d^3} \cdot \Delta_2$$

and

$$h_3 = \tfrac{1}{2} \cdot \frac{M_2}{M_1} \cdot \frac{R_1^4}{d^3} \cdot \Delta_2$$

and hence

$$h_1 = 2h_3 .$$

In the following tables the heights h_1 of the tides have been collected for different polytropic indices and different values of d :

Case I : Taking $R_1 = R_{\odot} = 7/1500$ astron. unit.

* Here n is quite different from the n used in §2 to §4 to denote $\frac{M_2}{M_1}$.

TABLE VI

d (astron. units).	$n=1$	$n=1.5$	$n=2$	$n=3$	$n=4$
1/30	$21.7\ 10^{-6}$	$18.7\ 10^{-6}$	$16.8\ 10^{-6}$	$15.9\ 10^{-6}$	$14.7\ 10^{-6}$
1/20	$6.2\ 10^{-6}$	$5.3\ 10^{-6}$	$4.7\ 10^{-6}$	$4.3\ 10^{-6}$	$4.2\ 10^{-6}$
1/10	$0.7\ 10^{-6}$	$0.6\ 10^{-6}$	$0.5\ 10^{-6}$	$0.48\ 10^{-6}$	$0.47\ 10^{-6}$
1/5	$0.9\ 10^{-7}$	$0.8\ 10^{-7}$	$0.7\ 10^{-7}$	$0.6\ 10^{-7}$	$0.6\ 10^{-7}$
3/10	$0.3\ 10^{-7}$	$0.26\ 10^{-7}$	$0.23\ 10^{-7}$	$0.20\ 10^{-7}$	$0.20\ 10^{-7}$
2/5	$0.1\ 10^{-8}$	$0.97\ 10^{-8}$	$0.86\ 10^{-8}$	$0.77\ 10^{-8}$	$0.73\ 10^{-8}$
1/2	$5.8\ 10^{-9}$	$4.9\ 10^{-9}$	$4.3\ 10^{-9}$	$3.9\ 10^{-9}$	$3.8\ 10^{-9}$
1	$0.7\ 10^{-9}$	$0.6\ 10^{-9}$	$0.5\ 10^{-9}$	$0.48\ 10^{-9}$	$0.47\ 10^{-9}$
5	$5.8\ 10^{-12}$	$4.9\ 10^{-12}$	$4.3\ 10^{-12}$	$3.9\ 10^{-12}$	$3.8\ 10^{-12}$
10	$0.7\ 10^{-12}$	$0.6\ 10^{-12}$	$0.5\ 10^{-12}$	$0.48\ 10^{-12}$	$0.47\ 10^{-12}$

TABLE VII

Case II : $R_1=2R_{\odot}$ and $R_2=4R_{\odot}$.

d (astron. units.)	$n=1$	$n=1.5$	$n=2$	$n=3$	$n=4$
1/30	$1.39\ 10^{-3}$	$1.19\ 10^{-3}$	$1.07\ 10^{-3}$	$1.02\ 10^{-3}$	$0.94\ 10^{-3}$
1/20	$0.40\ 10^{-3}$	$0.34\ 10^{-3}$	$0.30\ 10^{-3}$	$0.27\ 10^{-3}$	$0.27\ 10^{-3}$
1/10	$44.8\ 10^{-6}$	$38.4\ 10^{-6}$	$32.0\ 10^{-6}$	$30.7\ 10^{-6}$	$30.1\ 10^{-6}$
1/5	$57.6\ 10^{-7}$	$51.2\ 10^{-7}$	$44.8\ 10^{-7}$	$38.4\ 10^{-7}$	$38.4\ 10^{-7}$
3/10	$19.2\ 10^{-7}$	$16.6\ 10^{-7}$	$14.7\ 10^{-7}$	$12.8\ 10^{-7}$	$12.8\ 10^{-7}$
2/5	$70.4\ 10^{-8}$	$62.4\ 10^{-8}$	$55.0\ 10^{-8}$	$49.6\ 10^{-8}$	$46.4\ 10^{-8}$
1/2	$371.2\ 10^{-9}$	$313.6\ 10^{-9}$	$275.2\ 10^{-9}$	$249.6\ 10^{-9}$	$243.2\ 10^{-9}$
1	$44.8\ 10^{-9}$	$38.4\ 10^{-9}$	$32.0\ 10^{-9}$	$30.7\ 10^{-9}$	$30.1\ 10^{-9}$
5	$371.2\ 10^{-12}$	$313.6\ 10^{-12}$	$275.2\ 10^{-12}$	$249.6\ 10^{-12}$	$243.2\ 10^{-12}$
10	$44.8\ 10^{-12}$	$38.4\ 10^{-12}$	$32.0\ 10^{-12}$	$30.7\ 10^{-12}$	$30.1\ 10^{-12}$

From the above tables we know that the maximum tides are generated when $n=1$; hence we shall consider the case $n=1$ only.

Let the maximum heights of the tides generated in S_2 be given by h'_1 where

$$h'_1=\frac{M_1}{M_2}\cdot\frac{R_2^4}{d^3}\cdot\left|\Delta_2+\frac{R_2}{d}\Delta_3+\frac{R_2^2}{d^2}\Delta_4\right.$$

Now the distance between the distorted surfaces of the two stars would be zero, if

$$\begin{aligned} d &= R_1 + h_1 + R_2 + h'_1 \\ &= R_1 + R_2 + \frac{M_2}{M_1} \cdot \frac{R_1^4}{d^3} \cdot \left[\Delta_2 + \frac{R_1}{d} \Delta_3 + \frac{R_1^2}{d^2} \Delta_4 \right] \\ &\quad + \frac{M_1}{M_2} \cdot \frac{R_2^4}{d^3} \cdot \left[\Delta_2 + \frac{R_2}{d} \Delta_3 + \frac{R_2^2}{d^2} \Delta_4 \right] \end{aligned}$$

Knowing R_1, R_2, M_1, M_2 and Δ's we can determine d from this. We find that the distorted surfaces would touch when

Case I : $d = 1/100$ astronomical unit,

Case II : $d = 1/30$,, ,, .

Thus in the case I the tidal elongations of the distorted surfaces touch each other when the distance between the centres of the two stars is about $1/100$ astron. unit and in the case II when the distance between them is $1/30$ astron. unit. If the distance between them be less than what is mentioned above, there would be a colliding action between the materials of the two stars. In case I as we assumed the shortest distance between the stars to be $1/40$ astron. unit even at the perihelion of the relative orbit, the tides would not touch each other. In case II, the material contact or collision lasts as long as the stars remain within a distance of about $1/30$ astron. unit of each other, but it must be remembered that mere material contact does not ensure the conditions for ejection and disruption.

We shall now study the effect of the relative motion of the stars on the phenomenon of the tides. For the sake of simplicity we shall regard S_2 at rest and S_1 moving relative to S_2 for the time being. Here it is to be noted that the stars are assumed to be non-rotating.

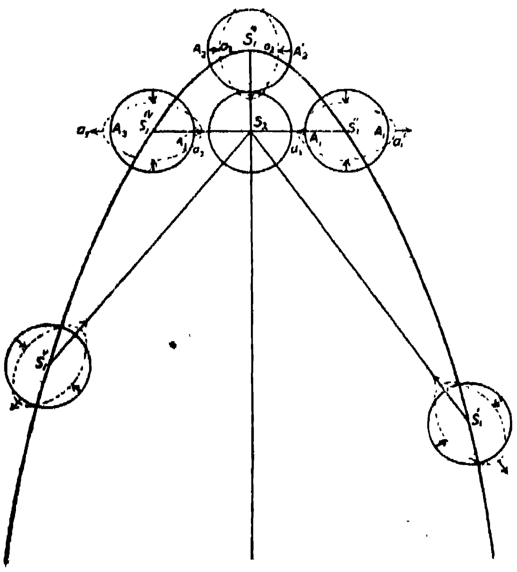


DIAGRAM III

In the diagram III S_1''' is the perihelion and S_1'' and S_1^{IV} are the ends of the latus-rectum. The complete circles represent the sections of the star's undisturbed surface by the relative orbital plane, whereas dotted figures represent the sections of the star's disturbed surface if in every position the tidal forces have got sufficient time to produce the tides.

For any possible formation of the planets, it is necessary only to study what becomes of the planetary ribbon when the stars begin to recede from each other after the closest approach.

During the short interval of a few hours (15 hrs. in Case I and 6.5 hrs. in Case II) the star S_1 moves from the position S_1'' to S_1^{IV} through S_1'''. A_1A_1', $A_2\,A_2'$, $A_3\,A_3'$ stand for the same diameter in the three positions of the star since we have taken the star to be non-rotating. During this time the tidal forces change directions two times (which may be noted at once by marking the direction of the tidal forces at any point such as A or A'/ and also the points of the maximum tides sweep over the whole section of the surface of S_1 by the relative orbital plane. When the star is at S_1'', A_1 is the point of the maximum tide and the material situated at A_1 moves towards a_1, but before the tidal material has moved over any considerable distance, S_1 moves to S_1''' and the direction of the tidal force at A_2 is reversed ; the matter is given acceleration in the direction exactly opposite to the previous one and the material at A_2 tends to move towards a_2 nullifying the previous motion. But before the material at A_2 moves any considerable distance towards a_2, the star S_1 moves to S_1^{IV}, once again changing the direction of the tidal forces. Thus we can see that the motion of the tidal material will be more or less oscillatory, but its amplitude would be much less than the maximum elongation possible under the circumstances. Thus it can be seen that as long as the star is between S_1'' and S_1^{IV}, there is little or no chance for the formation of the planetary ribbon.

As we have calculated above, when the distance between the stars is greater than 1/100 astron. unit in Case I and 1/30 astron. unit in Case II, even if it be possible for the tides to reach the maximum heights, they would not be able to touch each other and thus no planetary ribbon would be possible after the star has crossed the position S_1^{IV} as we have taken

$$S_2S_1^{IV} = 1/20 \text{ astron. unit.}$$

In order to satisfy the condition for the ejection of the material from S_1 sufficient for the formation of the planets we have to take S_2 to be very massive (rather unusually massive) as pointed out by the calculations in §2, §3 and §4. We have considered these two hypothetical cases because the previous workers (named in §1) have given much discussions to them. If we consider the masses which are required to ensure the conditions for the disruption and ejection of the necessary material the times calculated in these two cases are further diminished and the star S_1 takes much less time in moving from S_1'' to S_1^{IV}. Not only this, the actual hyperbolic motion would be much faster and the calculated times would be further decreased.

Thus there seems to be absolutely no possibility of the formation of the desired planetary ribbon by a close encounter or the grazing collision between the two stars of usual masses.

§6. Let us now examine the hypothesis mentioned in §1, *viz.*, " that our sun might have been a binary star having a companion.................. ....and that the collision between this body and a passing star broke the companion into fragments from which the present planets were developed." Once the conclusions of the two-body problem discussed above are accepted it would seem that the discussion of the following three-body problem is superfluous because the formation of the planetary ribbon is fundamental here also but a slight consideration would show that the nearness of the third star would alter the situation to a great deal and under the action of the sun and the intruding star the disruption of the companion star may be facilitated. Hence it is necessary to discuss this aspect of the tidal problem *de novo*.

Let us assume that the masses of the sun, its companion and the intruding star are M_0, M_1 and M_2 respectively and that G is the centre of mass of M_0 and M_1.

Let us choose our axes in space with fixed directions. Let the coordinates of M_0 relative to M_1 be (x, y, z) and those of M_2 relative to G be (x_1, y_1, z_1), then the motion of M_0 relative to M_1 will be given[8] by

$$\frac{d^2x}{dt^2}=\frac{M_0+M_1}{M_0M_1}\cdot\frac{\delta V}{\delta x};\quad \frac{d^2y}{dt^2}=\frac{M_0+M_1}{M_0M_1}\cdot\frac{\delta V}{\delta y};\quad \frac{d^2z}{dt^2}=\frac{M_0+M_1}{M_0M_1}\cdot\frac{\delta V}{\delta z}$$

and that of M_2 relative to G will be given by

$$\frac{d^2x_1}{dt^2}=\frac{M_0+M_1+M_2}{M_2(M_0+M_1)}\cdot\frac{\delta V}{\delta x_1};\quad \frac{d^2y_1}{dt^2}=\frac{M_0+M_1+M_2}{M_2(M_0+M_1)}\cdot\frac{\delta V}{\delta y_1};$$

$$\frac{d^2z_1}{dt^2}=\frac{M_0+M_2+M_2}{M_2(M_0+M_1)}\cdot\frac{\delta V}{\delta z_1},$$

where the gravitational potential V is given by

$$V=\frac{M_0M_1}{r_{0,1}}+\frac{M_1M_2}{r_{1,2}}+\frac{M_2M_0}{r_{2,0}},$$

and $r_{0,1}$ = distance between M_0 and $M_1=\sqrt{x^2+y^2+z^2}$,

$$r_{1,2}=\text{distance between } M_1 \text{ and } M_2=\sqrt{\sum_{x,y,z}\left(x_1+\frac{M_0x}{M_0+M_1}\right)^2},$$

$$r_{2,0}=\text{distance between } M_2 \text{ and } M_0=\sqrt{\sum_{x_1,y_1,z_1}\left(x_1-\frac{M_1x}{M_0+M_1}\right)^2}.$$

Here it is now evident that V is a function of $x, y, z; x_1, y_1, z_1$.

Therefore $dV = \sum_{x,y,z} \frac{\delta V}{\delta x} dx + \sum_{x_1, y_1, z_1} \frac{\delta V}{\delta x_1} dx_1.$

Hence $\frac{dV}{dt} = \frac{M_0 M_1}{M_0 + M_1} \sum_{x,y,z} \frac{d^2x}{dt^2} \cdot \frac{dx}{dt} + \frac{M_2(M_0+M_1)}{M_0+M_1+M_2} \sum_{x_1,y_1,z_1} \frac{d^2x_1}{dt^2} \cdot \frac{dx_1}{dt}.$

Integrating we have

$$\tfrac{1}{2}\frac{M_0 M_1}{M_0 + M_1} v^2 + \tfrac{1}{2}\frac{M_2(M_0+M_1)}{M_0+M_1+M_2} v'^2 = V + \text{constant}, \quad \ldots \quad (27)$$

where v is the velocity of M_0 relative to M_1 and v' is the velocity of M_2 relative to G. We have now to determine the constant.

Let us measure the time from the instant when M_2 was at an infinite distance from G and assume that the velocity of M_2 relative to G at that instant was v'_∞. Let us further assume that at that instant M_0 was describing a circular orbit relative to M_1 with a radius a. Therefore

$$\text{constant} = \tfrac{1}{2}\frac{M_2(M_0+M_1)}{M_0+M_1+M_2} v'^2_\infty - \tfrac{1}{2}\frac{M_0 M_1}{a},$$

since the distances $r_{2,0}$ and $r_{1,2}$ were infinite initially and the contribution to V by the terms involving these distances can be neglected.

Thus we have at any subsequent instant

$$\tfrac{1}{2}\frac{M_0 M_1}{M_0 + M_1} v^2 + \tfrac{1}{2}\frac{M_2(M_0+M_1)}{M_0+M_1+M_2} v'^2 - V = \tfrac{1}{2}\frac{M_2(M_0+M_1)}{M_0+M_1+M_2} v'^2_\infty - \tfrac{1}{2}\frac{M_0 M_1}{a}. \quad \ldots \quad (28)$$

Equation (28) may be regarded as the *energy equation*.

§7. After the collision between M_1 and M_2 is over, a ribbon is supposed to be formed between them. Luyten has suggested[9] that "the capture of this ribbon by the sun is not instantaneous and since the portions of the filament most favourable to capture are balanced precariously around the point of equality of the attractions of M_1 and M_2, it would seem that the capture cannot take place until the attraction of the sun on the filament is considerably greater than that of either of the parent stars." It is quite apparent from this that it is necessary for the sun to move roughly parallel to that portion of the filament which is most likely to be captured for considerable time. Now the filament would move in the plane of the relative orbit of M_1 and M_2 and hence the sun should also move in the same plane. This is not possible unless and until the orbital plane of M_0 relative to M_1 and that of M_2 relative to M_1 actually coincide before and after the collision. If we take the orbital plane of M_2 relative to M_1 as the x-y plane, then in the equation (28) the velocity v and v' reduce to velocities in the same plane and hence z is to be taken to be zero everywhere. It may also be mentioned that at the middle of the collision, the sun should move parallel to the middle of the filament and hence more or less parallel to the star M_2.

When M_2 has come considerably near to the binary system, the circular motion and the distance between the sun and its companion are bound to be disturbed. Let us assume that at some instant the distance between M_0 and M_1 is L and that it is moving with a velocity v relative to M_1 in the direction which makes an angle β with the straight line joining M_1 and M_2. Then the velocity of G at the instant relative to M_1 would be $\frac{M_0 v}{M_0 + M_1}$ in the same direction as that

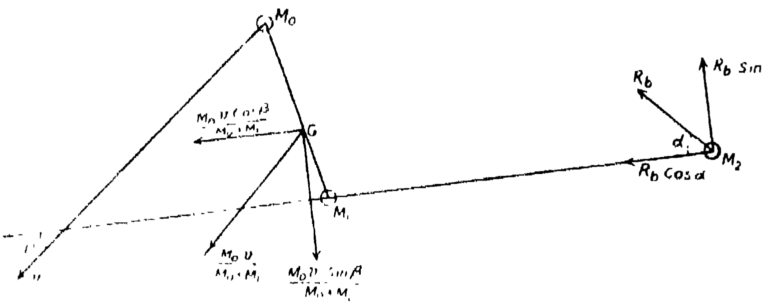


DIAGRAM IV

of the motion of M_0 (see diagram IV). Let us assume that at this instant M_2 is moving relative to M_1 with a velocity R_b and in the direction which makes an angle α with the straight line joining M_1 and M_2. Then the velocity of M_2 relative to G would be given by

$$v'^2 = R_b^2 + \left\{ \frac{M_0 v}{M_0 + M_1} \right\}^2 - 2\frac{M_0 R_b v}{M_0 + M_1} \cos(\alpha + \beta), \qquad \dots \quad (29)$$

where $\alpha + \beta$ is the angle between the directions of motions of G or M_0 and M_2, each relative to M_1.

As has been pointed out above, at the middle of the collision, M_0 must move parallel to M_2 and hence $\alpha + \beta = 0$ at that instant.

In order that the collision between M_1 and M_2 may take place, they have to come so near each other that the motion of M_2 relative to M_1 may be assumed to be solely conditioned by M_1 for some time before the collision. Suppose that when M_2 is at a distance l_1 from M_1, it begins to describe hyperbolic orbit relative to M_1 entirely under the influence of M_1 with a velocity $\overline{R_b}$ $\left(> \sqrt{\frac{2(M_0 + M_1)}{l_1}} \right)$. The equation to the orbit, the value of e and R_b, the velocity at any instant would be given by the equations (19), (18) and (20) respectively.

§ 8. Let us now investigate if it is possible for M_0 to continue its circular motion relative to M_1 right up to the instant when M_2 is at a distance l_1 from M_1 and is just beginning to describe a hyperbolic orbit relative to M_1 entirely under the influence of the latter. Now l_1 is so small that the attraction of M_1 on M_2 is appreciably greater than the attraction of M_0 on M_2. Also in order that M_2 may not collide with M_0 and that the hyperbolic motion of M_2 relative to M_1 may be possible, M_0 ought to be clearly out of its way, *i.e.*, ϕ, the angle between the line joining M_0 and M_2 and that joining M_2 and M_1 during this time ought to be greater than $\pi/2$ and less than $3\pi/2$, and the most favourable case is when $\phi=\pi$. In this case l_1 may be taken sufficiently great, say of the order a, at least. For definiteness we assume that $l_1=2a$.

Now α is the angle between the radius vector and the tangent to the hyperbolic orbit of M_2 relative to M_1 and hence is given by*

$$\tan\alpha=\frac{(e+1)\,d_1}{2ae\sin\alpha_0},$$

where α_0 is given by
$$2a=\frac{(e+1)d_1}{1+e\cos\alpha_0}$$

i.e.,
$$\sin\alpha_0=\sqrt{1-\left\{\frac{2a-(e+1)d_1}{2ae}\right\}^2}$$

and therefore
$$\tan\alpha=\frac{(e+1)d_1}{\sqrt{4a^2e^2-\{2a-(e+1)d_1\}^2}}.$$

We have $\beta=\frac{\pi}{2}$, as the orbit of M_0 is circular with respect to M_1 and

$$v^2=\frac{M_0+M_1}{a}$$

also
$$V=\frac{M_0M_1}{a}+\frac{M_2M_1}{2a}+\frac{M_2M_0}{3a},$$

since $\phi=\pi$.

Let v_1' be the value of v' in this position.

* Let the equation to the hyperbola be $r=\frac{l}{1+e\cos\theta}$. In Cartesian co-ordinates this reduces to $y^2-(e^2-1)x^2+2elx-l^2=0$. "$m$" of the tangent at a point $(a_1\cos\alpha_0, a_1\sin\alpha_0)$ is given by $m=\tan\theta=\frac{(e^2-1)a_1\cos\alpha_0-el}{a_1\sin\alpha_0}$. α, the angle between the radius vector and the tangent $=\theta-\alpha_0=\tan^{-1}\frac{\tan\theta-\tan\alpha_0}{1+\tan\theta\tan\alpha_0}=\tan^{-1}\frac{l}{a_1e\sin\alpha_0}$.

Substituting these values in (28), (29) and (20), we have

$$\frac{M_0M_1}{2a} + \frac{1}{2}\frac{M_2(M_0+M_1)}{M_0+M_1+M_2}v_1'^2 - \left\{\frac{M_0M_1}{a} + \frac{M_1M_2}{2a} + \frac{M_2M_0}{3a}\right\}$$

$$= \frac{1}{2}\frac{M_2(M_0+M_1)}{M_0+M_1+M_2}v_\infty'^2 - \frac{1}{2}\frac{M_0M_1}{a}$$

or $$\frac{1}{2}\frac{M_2(M_0+M_1)}{M_0+M_1+M_2}v_1'^2 = \frac{1}{2}\frac{M_2(M_0+M_1)}{M_0+M_1+M_2}v_\infty'^2 + \frac{M_1M_2}{2a} + \frac{M_2M_0}{3a} \quad \dots \quad (30)$$

and $$v_1'^2 = R_b^2 + \left\{\frac{M_0}{M_0+M_1}\right\}^2 \frac{M_0+M_1}{a} + \frac{2M_0}{M_0+M_1}R_b\sqrt{\frac{M_0+M_1}{a}}\sin\alpha, \quad \dots \quad (31)$$

as $\beta = \frac{\pi}{2}$,

where $$\tan\alpha = \frac{(c+1)d_1}{\sqrt{4a^2c^2 - \{2a-(c+1)d_1\}^2}}$$

or $$\sin\alpha = \frac{\sqrt{c+1}\,d_1}{2\sqrt{a^2(c-1)+ad_1}} \quad \dots \quad (32)$$

and $$R_b = \frac{M_1+M_2}{a} + \frac{M_1+M_2}{d_1}(c-1). \quad \dots \quad (33)$$

Knowing d_1, v'_∞, M_0, M_1 and M_2, we can calculate c from (30), (31), (32), and (33) and thence we can find out $\bar{R}_b$ and v'_1.

A critical study of these equations (30), (31), (32) and (33) shows that in ordinary circumstances there is absolutely nothing against assuming that right up to the commencement of the hyperbolic motion of M_2 relative to M_1 at a distance of $2a$ from M_1, M_0, which is describing a circular orbit relative to M_1, is moving with the velocity $\sqrt{\frac{M_0+M_1}{a}}$; ϕ may be taken to be π.

Now the problem reduces to the following stage :—

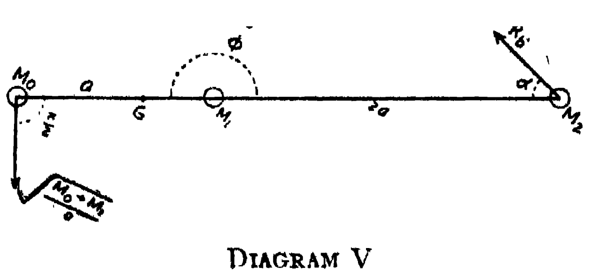


DIAGRAM V

M_0 is describing a circular orbit relative to M_1 with the velocity $\sqrt{\dfrac{M_0+M_1}{a}}$; and is situated on the straight line joining M_1 and M_2 as we have taken $\phi=\pi$. M_2 is at a distance $2a$ from M_1 and it just commences its hyperbolic motion relative to M_1 entirely under the influence of the latter with the relative velocity R_b; the eccentricity e of the relative orbit is known.

Now the theory requires that M_1 is ionized from M_0 due to the encounter and collision and hence at the middle of the collision M_0 must possess some velocity greater than the parabolic velocity at its position relative to M_1; for, at this instant, the tidal forces on M_1 reach their maximum and thus this situation is the most favourable for the ionization. We shall suppose that at the middle of the collision, M_0 has got a velocity equal to $\kappa\sqrt{\dfrac{2(M_0+M_1)}{a_1}}$ where $\kappa \geq$ and a_1 is the distance of M_0 from M_1 at that instant.

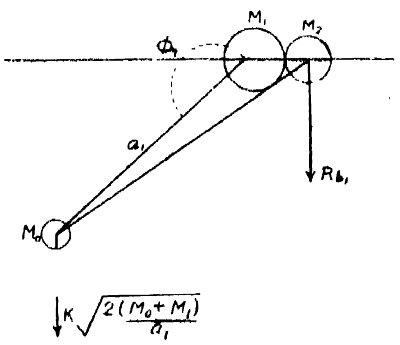


DIAGRAM VI

Now $$V=\frac{M_0M_1}{a_1}+\frac{M_1M_2}{d_1}+\frac{M_2M_0}{\sqrt{a_1^2+d_1^2-2a_1d_1\cos\phi_1}},$$

where ϕ_1 is the angle between the lines of centres of M_0, M_1 and M_1, M_2,

$R^2_{b_1}$ = square of the velocity of M_2 relative to M_1 in this position

$$=\frac{2(M_1+M_2)}{d_1}+\frac{(M_1+M_2)(e-1)}{d_1} \text{ from (20)} \qquad \dots (34)$$

$$v^2=\frac{2(M_0+M_1)}{a_1}\kappa \quad \text{(supposition)}, \qquad \dots (35)$$

$\alpha+\beta=0$,

v'^2_2 = square of the velocity of M_2 relative to G in this position

$$=R^2_{b_1}+\left[\frac{M_0}{M_0+M_1}\right]^2\frac{2(M_0+M_1)}{a_1}\kappa^2-\frac{2M_0}{M_0+M_1}R_{b_1}\kappa\sqrt{\frac{2(M_0+M_1)}{a_1}} \qquad \dots (36)$$

and $\frac{M_0M_1}{a_1}(\kappa^2-1)+\frac{1}{2}\frac{M_2(M_0+M_1)}{M_0+M_1+M_2}v_2'^2+\frac{M_0M_1}{2a}$

$$=\frac{M_1M_2}{d_1}+\frac{M_2M_0}{\sqrt{a_1^2+d_1^2-2a_1d_1\cos\phi_1}}+\frac{1}{2}\frac{M_2(M_0+M_1)}{M_0+M_1+M_2}v'^2_\infty \quad \dots (37)$$

from (28).

Making use of the value of ϵ, as determined previously, in (34), (36) and (37), we can obtain an equation involving M_0, M_1, M_2, κ, a_1, v'_∞, d_1 and a.

Assuming values for M_0, M_1, M_2, a, d_1 and v'_∞, we shall have an equation between κ and a_1. From this we can investigate if it is possible to have any value for a_1 of the order of 10 to 18 astronomical units (the distance between the sun and the major planets) for some reasonable value of $\kappa \geq 1$.

§9. NUMERICAL CALCULATIONS

Case I : Let us assume that

$M_0=M_1=M_2=M_\odot$ mass of the sun

$d_1=1/40$ astron. unit

$a=18$,, ,, ,,

$v'_\infty=70$ Km./sec. $=7/3$ units/sec.,

where 1 unit$=30$ Km.

Substituting these values in (30), (31), (32) and (33), we get

$$v'_1=\sqrt{\frac{67}{12}} \text{ units/sec.} \quad \dots (30')$$

$$\sin\alpha=\frac{\sqrt{e+1}}{1440\sqrt{e-1+\frac{1}{720}}}, \quad \dots (32')$$

$$R_b^2=80\left\{e-1+\frac{1}{720}\right\} \quad \dots (33')$$

and

$$e=\frac{769}{720}-\frac{\sqrt{e+1}}{4320\sqrt{80}}. \quad \dots (31')$$

Therefore $e=1.068$ (approx.)

and $\bar{R}_b=\frac{\sqrt{50}}{3}$ units/sec.

Making use of these values in (34), (36) and (37), we have

$$R_{b_1}=\sqrt{165.44} \text{ units/sec.} \quad \dots (34')$$

and

$$v_2'=\sqrt{165.44+\frac{\kappa^2}{a_1}-\frac{25.724}{\sqrt{a_1}}\kappa} \quad \dots (36')$$

Also $$(1.33\,\kappa^2-1)\frac{1}{a_1}-\frac{1}{a\pm.03}-\frac{8.58}{\sqrt{a_1}}\,\kappa+13.36=0 \qquad (37')$$

positive or negative sign being taken according as ϕ_1 is equal to π or 0.

Equation (37′) defines a relation between κ and a_1. The values of a_1 for different values of κ have been tabulated below :—

TABLE VIII

κ	With $+ve$ sign in (37′)	With $-ve$ sign in (37′)
1	$.0011 < a_1 < .0012$	$.042 < a_1 < .043$
	$.49 < a_1 < .50$	$.51 < a_1 < .52$
2	$1.08 < a_1 < 1.09$	$.033 < a_1 < .034$
	$.07 < a_1 < .08$	$1.10 < a_1 < 1.11$
3	$1.99 < a_1 < 2.00$	$2.04 < a_1 < 2.05$
	$.297 < a_1 < .298$	$.024 < a_1 < .025$

Case II : Let us assume that

$$M_0=M_\odot,\ M_1=2M_\odot,\ M_2=8M_\odot,$$
$$R_0=R_\odot,\ R_1=2R_\odot,\ R_2=4R_\odot$$

We have taken R_1 and R_2 such as to give sufficient densities to M_1 and M_2 so that ultimately a ribbon of sufficient mass may be formed by a mild collision between M_1 and M_2 (if this be possible). If they are taken large, then to produce a ribbon of desired mass, sufficiently deep collision will be required; here we may put

$$d_1=\tfrac{1}{38}\text{ astron. unit } (R_1+R_2>\tfrac{1}{38}\text{ astron. unit})$$

and $$v'_\infty=70\text{ km./sec.}=7/3\text{ units/sec.}$$

Then from (30), (31), (32) and (33), we have, as in the Case I,

$$e=1.014\text{ (approx.)}$$

and $$R_b=\sqrt{5.876}\text{ units sec.}$$

Hence we get

$$(2.73\kappa^2-2)\frac{1}{a_1}-\frac{8}{a_1\pm.03}+221=20.12\,\kappa\sqrt{\frac{6}{a_1}}, \qquad \cdots\ (37'')$$

the positive or negative sign being taken according as ϕ_1 is put equal to π or 0.

The values of a_1 for different values of κ, as obtained from (37″), are tabuated below :—

TABLE IX

κ	With $+ve$ sign in (37″)	With $-ve$ sign in (37″)
1	$0.0002 < a < .00021$	$.000 < a_1 < .0004$
	$0.093 < a_1 < 0.094$	$.0044 > a_1 < .0054$
		$.04 < a_1 < .05$
		$.12 < a_1 < .13$
2	$.008 < a_1 < .009$	$.037 < a_1 < .038$
	$.18 < a_1 < .19$	$.203 < a_1 < .204$
3	$a_1 = .03$	$.035 < a_1 < .036$
	$a_1 - .3$	$.38 < a_1 < .39$

§10. *Conclusion* :—In sections §2 to §5, we have seen that even in the most favourable situations, there is little or no possibility for the formation of the planetary ribbon due to the close encounter or the grazing collision between two stars of usual masses.

Tables VII and IX show that for every value of κ (from 1 to 3) the corresponding values of a_1 are very small. Of these the admissible values are only those which satisfy the equations of conservation of angular momentum also ; but whichever value be selected, at the middle of the collision between M_1 and M_2, the sun M_0 would come so very near to M_1, and hence to M_2 that a collision or a very close encounter can hardly be avoided.

In view of the above facts, we can say that no existing tidal theory can satisfactorily explain the origin of the solar system.

§11. In the end I wish to express my very respectful thanks to Prosessor A. C. Banerji for his keen interest during the preparation of this paper.

NOTE

A month after writing this paper, the important work of Dr. Spitzer was published in the December (1939) issue of the Astrophysical Journal. From astrophysical considerations Spitzer has proved that even if the desired planetary ribbon is assumed to be formed by a close encounter or a grazing collision between two stars, it will be diffused in space without giving birth to the planets. In the present paper the author has shown from purely dynamical considerations that no planetary ribbon can be formed. Thus from entirely different considerations, Dr. Spitzer's work also supports the fundamental conclusion of this paper that the existing tidal theories are unable to explain the formation of the solar system.

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